Theoretical Physics Institute

TPI-MINN-93/55-T
UMN-TH-1227/93
Technion-PH-93-41
November 1993

Lifetimes of Charmed Hadrons Revisited. Facts and Fancy

Talk at the Third Workshop on the Tau-Charm Factory, June 1993, Marbella, Spain

Boris Blok

Department of Physics
Technion – Israel Institute of Technology
Haifa 32000, Israel

and

Mikhail Shifman

Theoretical Physics Institute, University of Minnesota
Minneapolis, MN 55455, USA

Abstract

The problem of the hierarchy of lifetimes of charmed hadrons is reviewed. The QCD-based theory of preasymptotic effects in inclusive weak decays dating back to the beginning of the eighties is now entering its mature phase. Combining recent and old results we argue that the observed hierarchy reflects most intimate features of the hadronic structure. The problem of a wide spread of lifetimes of charmed hadrons is addressed. We speculate on what is to be expected from QCD to provide the observed pattern. A number of predictions is given for the hierarchy of lifetimes in the family of the beautiful hadrons.

1 Introduction

From early days of QCD it is well-known that the lifetimes of all particles H_Q containing a heavy quark Q and light quarks are the same and equal to that of a 'free' heavy quark in the limit $m_Q \to \infty$ (m_Q is the mass of the quark Q). At the same time the experimentally observed total widths in the charmed family differ by as much as an order of magnitude for D^+ and Ξ_c^0. This scatter clearly shows that $m_Q \sim 1.4$ GeV is far below the asymptotic domain, and the questions arise whether the preasymptotic effects are understood in the present-day QCD and what one should expect for the b quark family. These questions are addressed in the present talk. As we will see later, the hierarchy of the lifetimes presents a junction of different aspects of the theory of strong interactions. The two main elements are (i) a systematic expansion of amplitudes in the inverse mass of the heavy quark; (ii) estimates of matrix elements of local operators (appearing in the expansion) over the hadronic states H_Q. The first step, the construction of the m_Q^{-1} expansion, takes into account the quark-gluon dynamics at short distances and is clean in the sense that it is based on fundamental QCD and is, thus, model-independent (with a certain reservation, to be discussed below). The technology of m_Q^{-1} expansions experienced a dramatic development in recent years [1, 2]. The corresponding achievements are incorporated in our discussion. As for the hadronic matrix elements, they reflect the hadron structure at large distances. As usually, here the progress was limited, if at all, and our purpose is to outline some possible approaches and reveal the impact of this or that specific mechanism on the lifetime hierarchy.

The 'scientific' approaches to the issue of the lifetime hierarchy date back to the beginning of the eighties. It was pointed out that the total widths are related, by means of unitarity, to the imaginary part of certain forward scattering amplitudes [3]. This simple and rather obvious observation paved the way to a generalization of the Wilson operator product expansion (OPE) allowing one to represent all amplitudes of interest in the form of a series in m_Q^{-1} [4, 5, 6], an expansion essentially equivalent to what is called now the Heavy Quark Effective Theory (HQET) [7, 8] (for reviews see e.g. ref. [9]). Shortly after it was discovered that the terms behaving like powers of m_Q^{-1} should be supplemented by hybrid logarithms [10]. These logarithms are a routine part of the present-day HQET. The analysis of preasymptotic effects in charm and beauty combining both power series in m_Q^{-1} and the

hybrid logarithms has been first carried out in [5] in the mid-eighties. At those days the data on baryons were scarce even in the c quark family, and next to nothing was known about the lifetimes in the b family. It is not surprising, therefore, that the emphasis in these works was on the operators which differentiate between the D^+ and D^0. These operators necessarily involve the spectator light quark and have dimension 6 or higher. A set of dimension 6 operators was analyzed in [5, 6] [1] with definite predictions concerning the hierarchy of the lifetimes. Now, ten years later, it became clear that the analysis has to be updated for many reasons.

First, the numerical values of some crucial parameters turned out to be higher than previously thought. As a result, a typical scale of the preasymptotic 'corrections' becomes larger to the extent that the 'corrections' overshadow the leading term in the charmed family (so that, strictly speaking, they can not be called corrections anymore). Even the beauty family is still not quite in the asymptotic domain.

The enhanced role of the preasymptotic effects leads, in turn, to the necessity of including other operators, not fully considered in this problem previously. There are two operators of dimension 5 and one new operator of dimension 6. Understanding the ratio of the baryon-to-meson lifetimes is impossible without proper consideration of these operators.

As we will see below, for $m_c \sim 1.4$ GeV the convergence of the expansion is unexpectedly *so bad* that even operators of dimension 7 and higher must play an important role. Not much can be said about them at the moment, which means, of course, that all results referring to the c quark family are doomed to be valid at best qualitatively, if at all. We have gloomy suspicions, to be shared with everybody, that the QCD-based 'inclusive' approach will never become fully quantitative in the charmed family. In the b quark family the situation seems to be more favorable. Here the the expansion parameter m_b^{-1} is sufficiently small to guarantee a reasonable convergence of the m_Q^{-1} series, and limiting to operators of dimension 5 and 6 is quite reasonable.

Next, one can use symmetry arguments and models to estimate the hadronic matrix elements of these operators, establishing in this way a hierarchy of lifetimes.

This stage, evaluation of the matrix elements, is the most vulnerable point of the existing theory, its Achilles' heel. One must frankly admit that at least

[1] Ref. [6] ignores the existence of the hybrid logarithms.

some of the relevant matrix elements are not controlled theoretically today. Hence, one is forced to move onto swampy soil of models. We invoke a primitive quark model for orientation. What is the outcome? Honestly, it should be admitted that the charmed family is too light for serious theorists building $1/m_Q$ expansions. With some imagination, however, we still dare to say that the pattern of the charm lifetimes is understandable. And further efforts, both experimental and theoretical, are worthwhile – the observed hierarchy (once it is actually established) would imply certain relations between the hadronic matrix elements of the operators involved, a unique information, inaccessible in any other way.

Using the fact that this talk is not subject to the scrutiny of refereeing we allow ourselves to indulge in a fantasy treating the $1/m_c$ expansion and the primitive quark model mentioned in a rather speculative way. An element of wishful thinking is quite obvious in this talk, and we bring our apologies for this fact. Our only justification is a hope that the corresponding speculations will catalyze a more 'scientific' analysis in future.

We start with a necessary theoretical background and a brief historic review. Then we proceed to the picture of the c quark family. Finally, a few remarks concerning our expectations in the b quark family are given.

2 Theoretical Excursion

2.1 General picture

In prehistoric times (the seventies) the inclusive heavy flavor decays were treated in the most straightforward way – the width of the Q containing hadron H_Q was assumed to be equal to the probability of the free Q quark decay. An immediate consequence is the equality of the widths of all hadrons belonging to the Q family.

Although in the limit $m_Q \to \infty$ this prescription is certainly correct a single glance on what we actually have in the charmed family (see Table 1) shows how far the existing quarks are from the asymptotic domain and how important preasymptotic corrections are. Even leaving aside Ξ_c^0 where the data are rather poor and the error bars are large we can say that the lifetimes span the interval from roughly 10×10^{-13} sec for D^+ (the most long-living charmed particle) to 2×10^{-13} for Λ_c (see ref. [11] for the recent analysis of

the experimental data). There is a drastic imparity between D^+ and D^0 and between mesons and baryons. Do we understand this hierarchy theoretically?

The idea lying at the base of the modern theory of the preasymptotic effects is simple and can be compared to a well-known text-book problem. We mean the problem of the nuclear β decay. If the energy released in the β electron is much larger than a typical binding energy of both electrons in the shell and nucleons in the nucleus (the latter is unrealistic, of course) one can forget about the electrons in the shell and other bound state effects in calculating the decay probability. In the high precision calculations, however, the bound state effects should be accounted for. In particular, the soft electrons in the shell do interfere with the β electron (the Pauli exclusion principle), and this interaction generates corrections proportional to inverse powers of the energy release.

2.2 The inverse quark mass expansion

Likewise, in the heavy quark decay all participating degrees of freedom can be either hard (the corresponding energy is of order m_Q) or soft. In the leading approximation we ignore the soft degrees of freedom and treat the hard ones as free – a good old free quark decay (Fig. 1). The inclusion of the soft degrees of freedom in analysis generates power non-perturbative corrections. An obvious example of the soft degree of freedom is the spectator light quark. This is not the end of the story, of course. Soft gluons in the initial light cloud surrounding Q or those emitted by the final light quarks is another example (see Fig. 2).

The formal basis allowing one to systematically catalog the preasymptotic non-perturbative corrections is provided by Wilsonian operator product expansion (OPE). More exactly, what is readily calculable are the so called condensate non-perturbative effects.

Although the word OPE might sound frightening to non-experts, conceptually this is a very simple formalism, just a procedure of a systematic separation of large and short distance contributions in QCD (hard and soft degrees of freedom). We further assume that the short-distance contribution residing in the coefficient functions can be found perturbatively, while all non-perturbative effects are attributed to matrix elements of the operators appearing in the expansion. This procedure is sometimes called the practical version of OPE. We will refer to it as to the 'standard'. It is clear

that this standard prescription is by no means exact since in principle there are non-perturbative contributions even at short distances modifying the coefficient functions. (They will be referred to as hard or non-condensate non-perturbative terms). The latter are poorly controllable theoretically at present, and we will just assume that they can be disregarded for the time being. It seems quite probable, though, that they are non-negligible in the charmed family. Anyhow, we will disregard them. The 'hard' non-perturbative terms should be (and probably will be) targeted in future analyses of the preasymptotic effects in the charm decays.

Constructing the operator product expansion we obtain the inclusive decay probabilities as an expansion in m_Q^{-1}. Practically all terms up to m_Q^{-3} are known at present. We will discuss them and then speculate on higher order terms.

The statement above is an oversimplification. Apart from the power dependence on m_Q^{-1} one encounters logarithmic dependencies on this parameter – the so called hybrid anomalous dimensions [10] (matching logarithms of HQET). For pedagogical purposes let us forget for a while about the hybrid logs. We will comment on them later.

We start from the effective weak lagrangian describing non-leptonic decays

$$\mathcal{L}(\mu) = \frac{G_F}{\sqrt{2}} V_{sc} V_{ud} (c_1 \mathcal{O}_1 + c_2 \mathcal{O}_2) \tag{1}$$

where $c_{1,2}$ are Wilson coefficients accounting for the loop momenta from μ up to M_W and known in perturbation theory while $\mathcal{O}_{1,2}$ are operators,

$$\mathcal{O}_1 = (\bar{s}\Gamma_\mu c)(\bar{u}\Gamma_\mu d), \;\; \mathcal{O}_2 = (\bar{s}_i \Gamma_\mu c^j)(\bar{u}_j \Gamma_\mu d^i). \tag{2}$$

Here $\Gamma_\mu = \gamma_\mu(1+\gamma_5)$. Eqs. (1), (2) present the lagrangian relevant to the $c \to s\bar{d}u$ transition. The penguin graphs showing up at the 1% level are omitted. The coefficients $c_{1,2}$ are known in the leading log and next-to-leading approximations [12, 13];

$$c_1 = \frac{1}{2}(C_+ + C_-), \;\; c_2 = \frac{1}{2}(C_+ - C_-). \tag{3}$$

In the leading approximation

$$C_\pm = \left[\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right]^{d_\pm}. \tag{4}$$

The numerical values of the coefficients depend on the choice of parameters. The physical quantities, like the non-leptonic widths, can not depend on the arbitrary scale μ. The μ dependence explicit in the weak lagrangian (1) is compensated in the process of calculation of Γ's. In the leading log approximation we limit ourselves to one can set $\mu = m_c$ from the very beginning. As for $\Lambda_{\rm QCD}$ the current fashion bends towards a rather high value, $\Lambda_{\rm QCD} \sim 300$ MeV [14]. Our gut feeling is that the genuine value of $\Lambda_{\rm QCD}$ is actually smaller, but this is no time to indulge in the discussion of this issue. Accepting this estimate we get

$$C_+ \sim 0.74, \;\; C_- \sim 1.8. \tag{5}$$

Now, we construct the transition *operator* $\hat{T}(c \to X \to c)$,

$$\hat{T} = i \int d^4x T\{\mathcal{L}(x)\mathcal{L}(0)\} = \sum_i C_i \mathcal{O}_i \tag{6}$$

describing a diagonal amplitude with the heavy quark c in the initial and final state (with identical momenta). The transition operator $\hat{T}$ is built by means of OPE as an expansion in local operators $\mathcal{O}_i$. The lowest-dimension operator in $T(c \to X \to c)$ is $\bar{c}c$, and the complete perturbative prediction corresponds to the perturbative calculation of the coefficient of this operator. In calculating the coefficient of $\bar{c}c$ we treat the light s, d, u quarks in X as hard and neglect the soft modes. Say, we ignore the fact that in a part of the phase space the u quark line is soft and can not be treated perturbatively. Likewise, we ignore interaction with the soft gluons.

Our task – the task of the theory of preasymptotic corrections – is the analysis of the influence of the soft modes in the quark and gluon fields manifesting themselves as a series of high-dimension operators in $\hat{T}$.

Once the expansion (6) is built we average $\hat{T}$ over the hadronic state of interest, charmed mesons and baryons in the case at hand. At this stage the non-perturbative large distance dynamics enters through the matrix elements of the operators of dimension 5 and higher. (There are no operators of dimension 4).

Finally, the imaginary part of $< H_c|\hat{T}|H_c >$ presents the H_c width sought for,

$$\Gamma(H_c) = \frac{1}{M_{H_c}} < H_c|{\rm Im}\hat{T}(c \to X \to c)|H_c > . \tag{7}$$

For experts we hasten to add a remark concerning peculiarities of the kinematics of the amplitude under consideration. It is essentially Minkowskian. Still the operator product expansion originally formulated for deep Euclidean kinematics can be used. It is important that we take the full imaginary part corresponding to all possible cuts of the diagrams (totally inclusive final state X). Omitting some of the cuts would lead to infrared unstable results [15]. Since the decay rates we calculate present integrated quantities over the interval of energies up to m_c the corresponding prediction for Γ's, through analyticity, is related to the calculation of the OPE coefficients with the characteristic off-shellness $\sim m_c$. For comparison let us mention a similar analysis of the τ decays [16].

2.3 Catalog of relevant operators

The leading operator in the expansion of $\hat{T}$ is

$$\mathcal{O}_0 = \bar{c}c. \tag{8}$$

Keeping only $\bar{c}c$ is equivalent to perturbative calculation of the coefficients $c_{1,2}$ followed by a perturbative calculation of the decay width Γ through the relation

$$\Gamma^{pert} = \frac{1}{m_c}\mathrm{Im} < c|\hat{T}|c >, \tag{9}$$

where taking a 'mathematical' matrix element over the c quark singles out the operator $\bar{c}c$.

It is worth noting that the contribution of $\mathcal{O}_0$ in the physical width Γ (not to be confused with Γ^{pert}!) is given by

$$\Gamma = 2\mathrm{Im}C_0 \frac{1}{2M_{H_c}} < H_c|\mathcal{O}_0|H_c > .$$

The matrix element $(1/2M_{H_c}) < H_c|\mathcal{O}_0|H_c >$ is 1 plus $\mathcal{O}(m_c^{-2})$ corrections (see eq. (12) below).

The next operator in our catalog is

$$\mathcal{O}_G = \frac{i}{2}\bar{c}\sigma_{\mu\nu}G_{\mu\nu}c \to -\bar{c}\vec{\sigma}\vec{B}c, \tag{10}$$

with dimension 5. This operator as well as $\mathcal{O}_\pi$, see below, generates a difference between mesons on one hand and baryons, on the other.

Let us make a side remark concerning another dimension 5 spin 0 operator,

$$\mathcal{O}_\pi = \bar{c}[D^2 - (vD)^2]c, \tag{11}$$

where v is the four-velocity of the hadron H_c. This operator does not appear explicitly in the calculation of the total widths due to the fact that $\mathcal{O}_\pi$ is not Lorentz scalar. On the other, the relation [1, 17]

$$\bar{c}c = \bar{c}\gamma_\mu v_\mu c - \frac{1}{2m_c^2}\mathcal{O}_\pi + \frac{1}{2m_c^2}\mathcal{O}_G + \frac{1}{4m_c^3}g^2\bar{c}\gamma_0 t^a c\sum_q \bar{q}\gamma_0 t^a q + O(1/m_c^4) \tag{12}$$

used in the calculation of the matrix element $< H_c|\bar{c}c|H_c >$ contains $\mathcal{O}_\pi$. (Here t^a are the color generators and q is a generic notation for light quarks).

We pass now to the $\mathcal{O}(m_c^{-3})$ terms due to operators of dimension 6. On general grounds one can limit the number of these operators to a few following structures [17].

(i) *Four-quark operators*

These have the generic form

$$\mathcal{O}_{4q} = (\bar{c}\Gamma q)(\bar{q}\Gamma c)$$

where q is a light quark field and Γ presents, in the case at hand, a combination of the Lorentz and color matrices. The coefficients of the four-quark operators have been first calculated in refs. [4, 5, 6]. They are determined by the one-loop graphs of the type presented on Fig. 3.

The four-fermion operators are responsible for the splittings between D^+ and D^0 and the splittings between different charmed baryons.

(ii) *Dimension 6 quark-gluon operators*

Apart from the heavy quark fields c and $\bar{c}$ relevant operators contain the gluon field strength tensor $G_{\mu\nu}$ (they may also include the covariant derivatives). These operators are generated by two-loop graphs (fig. 2) and, hence, their coefficients are numerically strongly suppressed compared to the four-quark case above.

By using equations of motion it is not difficult to show [17] that there are only two options for the spin-zero quark-gluon operators of dimension 6, namely

$$\bar{c}(D_\mu G_{\mu\nu})\Gamma_\nu c$$

and

$$\mathcal{O}_E = \bar{c}\sigma_{\mu\nu}G_{\mu\rho}\gamma_\rho iD_\nu c \to \bar{c}\vec{\sigma}\,\vec{E}\times i\vec{D}c, \tag{13}$$

where $\vec{E}$ is the chromoelectric field. All other operators of dimension 6 with the gluon field are reducible to the above. The operator $\bar{c}(D_\mu G_{\mu\nu})\Gamma_\nu c$ is actually a four-quark operator since

$$D_\mu G_{\mu\nu} = -g^2 \sum \bar{q}\gamma_\nu T^a q.$$

Its coefficient contains extra α_s/π, however, compared to the four-quark operators coming from the one-loop graphs, and the corresponding contribution can be neglected.

As for $\mathcal{O}_E$ its contribution turns out to be small numerically compared to that of $\mathcal{O}_G$, so that it plays no role in what follows. We mention it for completeness and will essentially forget about $\mathcal{O}_E$ from now on.

For pedagogical purposes it is instructive to write down the decomposition of the operators above in terms of the HQET field,

$$h_c(x) = e^{-im_c vx}\frac{(I+\hat{v})}{2}c(x), \tag{14}$$

although, certainly, this decomposition adds nothing in practical terms. In eq. (14) c is the normal Dirac bispinor, and v is the four-velocity of the heavy hadron; $\hat{v} \equiv \gamma^\alpha v_\alpha$.

Then, say, for $\bar{c}c$ we essentially repeat eq. (12),

$$\mathcal{O}_0 \to \bar{h}_c\gamma_\mu v_\mu h_c - \frac{1}{2m_c^2}\bar{h}_c\vec{\pi}^2 h_c - \frac{1}{2m_c^2}\bar{h}_c\vec{\sigma}\vec{B}h_c + ... \tag{15}$$

and

$$\mathcal{O}_G \to -\bar{h}_c\vec{\sigma}\vec{B}h_c + ... \tag{16}$$

It is possible to prove that all dimension 6 operators appearing in eqs. (15), (16) can be reduced to four-quark operators suppressed by α_s. They can be consistently discarded.

2.4 Coefficients

The operators listed above appear in $\hat{T}$ each with its own coefficient. Practically all of them have been calculated previously – for $\mathcal{O}_0$ in the prehistoric times. The coefficients of the operators of dimension 5 were found in the recent works [1, 2] and for the spectator-sensitive operators of dimension 6

in refs. [4, 5]. We supplement this set by a new spectator-blind operator of dimension 6, and consider a new source of $1/m_Q$ corrections – the expansion of matrix elements of operators of dimensions 3 and 5 in terms of $1/m_Q$.

The prehistoric coefficient C_0 is obviously equal to

$$\mathrm{Im}C_0 = \frac{1}{2}(3\Gamma_0)\eta \tag{17}$$

where

$$\Gamma_0 \equiv \frac{G_F^2 m_c^5 |V_{sc}|^2}{192\pi^3} \tag{18}$$

is a very convenient unit for all widths to be discussed here. The factor η reflects the hard gluon exchanges hidden in $C_\pm$,

$$\eta = \frac{C_-^2 + 2C_+^2}{3} \approx 1.5. \tag{19}$$

If the hard-gluon exchanges are accounted for in the next-to-leading order

$$\eta \to \eta J$$

where the explicit expression for J is given in ref. [18], see also [13]. Since the picture of the lifetime hierarchy we are aimed at is qualitative at best we limit ourselves to the leading approximation.

The strange quark mass is neglected, here and below, systematically. Since we deal exclusively with the current quarks (there is no such notion as the constituent quark in the QCD-based approach),

$$m_s^2/m_c^2 \sim 10^{-2}$$

and the approximation $m_u = m_d = m_s = 0$ in calculating the *inclusive* widths seems to be not bad. One should be alert, however – possible deviations from this limit might be larger than expected, and an estimate of the effects due to non-vanishing M_K^2 is definitely in the short list of questions for further investigation.

Now we are approaching a more interesting and less trivial stage of the analysis – we pass to soft modes. (A reminder: we discard non-condensate non-perturbative effects. This is not because they are necessarily small in the charm decays but because we do not know how to treat them. Thus, here

is another item from the short list – hopefully, a courageous theorist can be found for its investigation.) The first effect to show up is that due to the chromomagnetic operator $\bar{c}\vec{\sigma}\vec{B}c$ describing the correlation of the heavy quark spin in H_c with the chromomagnetic field in the light cloud surrounding c. Its dimension is 5, hence the corresponding contribution in Γ is $\mathcal{O}(m_c^{-2})$.

The chromomagnetic field $\vec{B}$ appears because both the initial quark c and the light fast quarks produced in the decay are coupled to it – they do not live in the empty space but, rather, in a medium, the 'brown muck' around the heavy quark (we prefer to call this medium 'light cloud'). There are two sources of $\mathcal{O}_G$: the direct coupling to $\bar{d}$ and the coupling to c. In the latter case $\mathcal{O}_G$ appears through the equations of motion for the c field. We will not dwell on the issue referring to the original publications [1, 2, 19] for further details. To the leading order in α_s the coefficients C_G and C_π are

$$C_G = -\Gamma_0 \frac{1}{m_c^2}(8c_+^2 - 2c_-^2), \tag{20}$$

$$C_\pi = 0. \tag{21}$$

$\mathcal{O}_\pi$ appears only when we take the matrix element of $\mathcal{O}_0$, along with an additional piece with $\mathcal{O}_G$, see eq. (12).

Although both operators, $\mathcal{O}_G$ and $\mathcal{O}_\pi$, are of dimension 5 their matrix elements over H_c may contain also corrections $\mathcal{O}(m_c^{-1})$ corresponding to $\mathcal{O}(m_c^{-3})$ terms in Γ's which turn out to be insignificant numerically.

We now move to the operators of dimension 6. As we argued above there are only two classes of operators that we have to consider here. All other operators can be reduced (through equations of motion) to color blind four-quark operators, whose Wilson coefficients are additionally suppressed by α_s. Since considering α_s corrections to the Wilson coefficients is beyond our accuracy now we shall neglect them. We are left with two classes of operators. First, and most important, are spectator-sensitive four-quark operators, whose analysis was carried out in refs. [4, 5, 6]. Their coefficients are numerically enhanced since they are generated by one-loop graphs. These operators lead to the lifetime differences between different mesons and different baryons. There are three distinct physical mechanisms summarized by the four-quark operators: annihilation, interference and quark-quark scattering in the baryons. Technically the distinction manifests itself in the fact that each of the three light quark lines, u, d or s can be soft. Pictorially

the softness of the line is depicted as follows: instead of drawing a solid line we just cut the soft line to remind that no perturbative expression can be used for this line. Three relevant graphs are given on fig. 3 a,b,c. The full contribution of the 4-quark operators to the transition operator is

$$\hat{T}_{4q} = \mathcal{L}_d + \mathcal{L}_u + \mathcal{L}_s. \tag{22}$$

Here

$$\mathcal{L}_d = 48\pi^2\Gamma_0 \frac{1}{m_c^3}(K_1(\bar{c}\Gamma_\mu c)(\bar{d}\Gamma_\mu d) + K_2(\bar{c}\Gamma_\mu d))(\bar{d}\Gamma_\mu c)), \tag{23}$$

$$\begin{aligned} \mathcal{L}_u &= 48\pi^2\Gamma_0 \tfrac{1}{m_c^3}(K_3(\bar{c}\Gamma_\mu c - \tfrac{2}{3}\bar{c}\gamma_\mu\gamma_5 c)(\bar{u}\Gamma_\mu u) \\ &+K_4(\bar{c}_i\Gamma_\mu c_k - \tfrac{2}{3}\bar{c}_i\gamma_\mu\gamma_5 c_k)(\bar{u}_k\Gamma_\mu u^i)), \end{aligned} \tag{24}$$

$$\begin{aligned} \mathcal{L}_s &= 48\pi^2\Gamma_0 \tfrac{1}{m_c^3}(K_5(\bar{c}\Gamma_\mu c - \tfrac{2}{3}\bar{c}\gamma_\mu\gamma_5 c)(\bar{s}\Gamma_\mu s) \\ &+K_6(\bar{c}_i\Gamma_\mu c_k - \tfrac{2}{3}\bar{c}_i\gamma_\mu\gamma_5 c_k)(\bar{s}_k\Gamma_\mu s^i)), \end{aligned} \tag{25}$$

where

$$\begin{aligned} K_1 &= (c_+^2 + c_-^2)/2, \\ K_2 &= (c_+^2 - c_-^2)/2, \\ K_3 &= -(c_+ + c_-)^2/8, \\ K_4 &= -(5c_+^2 + c_-^2 - 6c_+c_-)/8, \\ K_5 &= -(c_+ - c_-)^2/8, \\ K_6 &= -(5c_+^2 + c_-^2 + 6c_+c_-)/8. \end{aligned} \tag{26}$$

The term $\mathcal{L}_d$ describes in the case of the D^+ meson the destructive interference between the $\bar{d}$ quark produced in the charmed quark decay and the spectator $\bar{d}$ quark. The same operator in the case of Λ_c describes the $cd \to us$ scattering. The operators $\mathcal{L}_u$ and $\mathcal{L}_s$ describe the interference between the spectator quarks in baryons and the u or s quarks produced in the decay (this interference is destructive for the case of the u quark and constructive for the case of the s quark).

2.5 Hybrid anomalous dimensions

Above the coefficients were presented for the operators normalized at m_c. Generally speaking, these operators have the hybrid anomalous dimensions

[10, 5] reflecting the effect of the virtual momenta from m_c to a typical hadronic scale μ. If we know the hadronic matrix elements of the operators normalized at m_c we do not need to include the hybrid logs explicitly, they are already there. As we will see shortly this is the case with the operator $\mathcal{O}_G(\mu = m_c)$ whose matrix element over mesons is expressible in terms of the measured mass differences while the matrix elements over Λ_c, and Ξ_c vanish. The operators $\bar{c}c$ and $\bar{c}\vec{D}^2 c$ have zero hybrid anomalous dimensions.

As for the four-fermion operators, however, we have no independent information on their matrix elements. Thus we have to use models – vacuum saturation in mesons and quark models in baryons. The question arises as to what choice of the normalization point would ensure the best possible validity of these models.

As far as the strong interactions are concerned, m_c is a completely foreign parameter. Therefore, it is absolutely natural to think that the models work best for a low normalization point, $\mu \sim$ a few hundred MeV. Then one should evolve the four-fermion operators down to μ, include explicitly the hybrid anomalous dimensions and only then apply the models. The effect of the hybrid logarithms is known in the leading-log approximation [5].

Taking account of the hybrid logs changes the coefficients K_i introduced in eq. (26) as follows,

$$K_i \to K_i \eta_i \tag{27}$$

where

$$\begin{aligned}
K_1\eta_1 &= (c_+^2 + c_-^2 + \tfrac{1}{3}(1-\sqrt{\kappa})(c_+^2 - c_-^2))/2, \\
K_2\eta_2 &= \sqrt{\kappa}(c_+^2 - c_-^2)/2, \\
K_3\eta_3 &= -[(c_+ + c_-)^2 + \tfrac{1}{3}(1-\sqrt{\kappa})(5c_+^2 + c_-^2 - 6c_+c_-)]/8, \\
K_4\eta_4 &= -\sqrt{\kappa}(5c_+^2 + c_-^2 - 6c_+c_-)/8, \\
K_5\eta_5 &= -[(c_+ - c_-)^2 + \tfrac{1}{3}(1-\sqrt{\kappa})(5c_+^2 + c_-^2 + 6c_+c_-)]/8, \\
K_6\eta_6 &= -\sqrt{\kappa}(5c_+^2 + c_-^2 + 6c_+c_-)/8.
\end{aligned} \tag{28}$$

Here

$$\kappa = \alpha_s(\mu)/\alpha_s(m_c).$$

The evolution from m_c down to μ also adds new structures, for instance,

$$\Delta\mathcal{L}_d = 16\pi^2 \frac{1}{m_c^3} \Gamma_0 (c_+^2 - c_-^2)\kappa^{1/2}(\kappa^{-2/9} - 1)(\bar{c}\Gamma_\mu t^a c) j_\mu^a, \tag{29}$$

$$\Delta\mathcal{L}_s = -8\pi^2 \frac{\Gamma_0}{m_c^3}\kappa^{1/2}(\kappa^{-2/9}-1)(5c_+^2+c_-^2)(\bar{c}\Gamma_\mu t^a c - \frac{2}{3}\bar{c}\gamma_\mu\gamma_5 t^a c)j_\mu^a. \quad (30)$$

Here

$$j_\mu^a = \bar{u}\gamma_\mu t^a u + \bar{d}\gamma_\mu t^a d + \bar{s}\gamma_\mu t^a s.$$

The latter structures are similar to the penguin contributions in the weak charm decays. Since we neglected the penguins, it seems logical to discard the structures of the type (29), (30) as well. We refer the reader to ref. [5] for a detailed discussion of the hybrid logarithms in the transition operator.

Apart from the logarithmic renormalization of the currents and operators discussed above, hybrid logarithms also appear in the matrix elements of the 4-quark operator when we use the factorization approximation to calculate them. We shall return to this point in the next section.

Finally let us note that although the dimension 5 chromomagnetic operator $\mathcal{O}_G$ has non-zero hybrid anomalous dimension, we do not have to take into account the hybrid renormalization when we analyses its contribution. The reason is that the matrix element of $\mathcal{O}_G$ normalized at m_c will be expressed in the next section through physical quantities – the masses of the charmed hadrons .

3 The hierarchy problem

The most striking feature of the experimental data on the lifetime hierarchy obvious from the first glance at Table 1 is its wide spread, especially for the case of the baryons. This directly contradicts our intuition. Indeed, the known source of the differences in lifetimes between different baryons are the 4-quark operators. Although their coefficients are enhanced by a factor $4\pi^2$ their matrix elements are suppressed by a factor $f_D^2/M_D^2 \sim 0.01$. (Here f_D is the value of the decay constant of the D meson). Hence, one could expect the lifetime spread in the hierarchy to be at most of order one. Certainly one could not forsee that the lifetimes will span an order of magnitude, as experimental data show. The lifetimes differing by 50 to 100% was the prediction of the first works on this subject [4, 5]. In addition to this, the analysis [4, 5] had other problems. For instance, it was obtained that the lifetimes of Λ_c and Ξ_c^+ are the same irrespective of the absolute value of $< \mathcal{O}_{4q} >$. This result seems to contradict the experimental data. Some progress in understanding of the latter problem was achieved in ref. [6], where

it was noticed that the relative weights of different 4-quark operators are very sensitive to the Wilson coefficients c_+ and c_-. The use of the modern value of these coefficients corresponding to $\Lambda_{\rm QCD} \sim 250 - 300$ MeV substantially improves the situation with Λ_c/Ξ_c^+ compared to the earlier results of ref. [5]. Still the problem of the wide spread remains. Can we explain it from first principles? The remainder of this talk will be devoted to an explanation (or, more precisely to speculations) why such a hierarchy can be expected from QCD (or, more precisely, how QCD must work so that the theory will be able to explain the experimental data).

4 What Can Be Said about Mesonic and Baryonic Matrix Elements

Unfortunately, the issue of the matrix elements, the most underdeveloped element of the whole procedure at present, can not be deferred indefinitely. We have to address it, realizing, of course, that now we are not going to enjoy the same degree of theoretical control we had previously.

For mesons the situation is not that bad, though, as will be seen below. At the same time in baryons we have to use much more shaky models.

For each operator in the OPE we have discussed we consider first mesonic matrix elements and then, later on, the matrix elements over the baryon states.

4.1 Mesons

The first operator of dimension 5 is the operator $\bar{c}\sigma Gc$. The matrix elements of this operator between the mesonic states were discussed in detail in ref. [20]. Remarkably, this matrix element can be determined in a model independent way, using the HQET symmetries:

$$\frac{1}{4M_D} < D|\bar{c}i\sigma Gc(\mu = m_c)|D > \equiv \mu_G^2 = \frac{3}{4}(M^2(D^*) - M^2(D)) \sim 0.4\mathrm{GeV}^2. \tag{31}$$

The next relevant matrix element is the operator of the kinetic energy

$$-\frac{1}{2M_D} < D|\bar{c}\vec{D}^2c|D > \equiv \mu_\pi^2 \sim 0.5\mathrm{GeV}^2. \tag{32}$$

Here we used the QCD sum rule results [21].

We now move to the dimension 6 operators. Just one remark to demonstrate that $\mathcal{O}_E$ does not play a role. Indeed, assuming that the chromoelectric field is of the same order as the chromomagnetic field, we obtain that the matrix element of this operator is suppressed by a factor $\sim \mu_\pi/m_c \sim 0.4$ in comparison with the contribution of dimension 5 operators. The corresponding Wilson coefficient is

$$c_E = 4\Gamma_0/m_c^3$$

The contribution of this dimension 6 operator is less than 10% of the contribution of the dimension 5 operators and can be safely neglected.

Let us turn now to the four-quark operators. The simplest approach allowing one to estimate the matrix elements of these operators is to use factorization. Consider first the operator $\bar{c}\Gamma_\mu d\bar{d}\Gamma_\mu c$. Note first that this matrix element contributes only to the D^+, but not to the D^0 meson. This matrix element can be estimated using factorization, or the vacuum saturation method:

$$< D^+|\bar{c}\Gamma_\mu d\bar{d}\Gamma_\mu c|D^+ >= f_D^2 M_D^2 \tag{33}$$

where f_D is the D meson decay constant,

$$< 0|\bar{c}\Gamma_\mu d|D >= f_D i p_\mu. \tag{34}$$

The operator $\bar{c}\Gamma_\mu d\bar{d}\Gamma_\mu c$ and f_D are both normalized at one and the same point, m_c. If we would like to proceed to f_D normalized at μ the corresponding hybrid logarithm should be inserted.

We pause here to make a remark which may be very important numerically. The question is which coupling f_D is to be substituted in eq. (33). The physical coupling f_D contains in itself corrections m_c^{-1} that are known to be very significant. Therefore, if we want to have a systematic $1/m_c$ expansion we must use a static value of f_D, with the $1/m_c$ terms removed. The difference between the physical and the static value of f_D clearly manifests itself in our expansion for the widths as an effect $\mathcal{O}(m_c^{-4})$. Were the expansion well convergent whatever value is substituted would not matter. For the actual c quark this matters a lot, since the physical value of f_D is ~ 170 MeV, while the static $F_D \sim 400$ MeV, i.e. $\sim$2.5 times larger than the physical one [22, 23]. The difference between f_D^2 and F_D^2 is humongous. The static coupling constant is defined as follows

$$< 0|\bar{d}\Gamma_\mu h|D >= iF_D v^\mu \sqrt{M_D/2}. \tag{35}$$

We will use capital F to emphasize the distinction between the physical and the static constants. F_D defined by eq. (35) depends on the mass of the heavy quark as

$$F_D \sim |\psi(0)|^2/\sqrt{m_c} \tag{36}$$

modulo hybrid logs. Here $\psi(0)$ is the value of the heavy meson wave function at the origin that does not depend on the mass of the heavy quark. If the value of matrix element (33) calculated with F_D is substituted in our OPE we see that the corresponding contribution is pure $1/m_c^3$ compared to perturbation theory.

On the other hand, the leptonic decay constant f_D defined using eq. (34) contains all orders of $1/m_c$. Indeed, the relativistic Dirac field $c(x)$ describing the c quark in QCD is connected with the the c quark field in HQET as

$$c(x) = e^{-im_c vx} \Sigma_k \frac{(i\hat{D})^k}{(2m)^k} h_c. \tag{37}$$

Here $\hat{D} \equiv \gamma^\alpha D_\alpha$. Consequently, the static and physical decay constant are connected as

$$f_D = F_D(1 + C_1/m_c + ...), \tag{38}$$

C_1 is a constant independent of the heavy quark mass. It can be (and was) determined in the QCD sum rules [22] and on the lattices [23]. It turns out that the contribution of the term proportional to C_1 in eq. (38) is approximately 50% of the contribution of the leading term and has the negative sign.

Only a systematic analysis of all dimension 7 operators can tell us what value – physical versus static – is more relevant to the problem. In the absence of such an analysis we will just speculate without even pretending on seriousness.

Other four-quark matrix elements can be determined also using factorization and the Fierz identities. In particular,

$$< D|\bar{c}\Gamma_\mu t^a d\bar{d}\Gamma_\mu t^a c|D > \approx 0. \tag{39}$$

For the product of two color-singlet brackets factorization becomes exact in the limit of the infinite number of colors.

The natural question is how well factorization works in the real world. The corresponding estimate is completely analogous to the estimates of the

$\bar{D}-D$ mixing parameters where factorization works with the accuracy of order 20%. This means that for mesons we can determine matrix elements in a model independent way with a sufficiently high accuracy.

4.2 Baryons

We now move to baryons. Consider first the dimension 5 operators. We start from the operator $\bar{c}\sigma Gc$. The matrix elements of this operator between the members of the antitriplet of the charmed baryons are zero. Indeed, this matrix element is proportional to correlation between the spin of the heavy quark and that of the light cloud. The latter spin in the baryons – members of the antitriplet is zero.

An estimate of this matrix element for the baryon Ω_c belonging to the sextet of the charmed baryons (and so the light cloud there has spin 1) can be carried out in the same way as for mesons. Recall that a part of the Hamiltonian of the heavy quark effective theory responsible for the spin splittings between the hadrons has the form

$$\Delta H_{\rm eff} = \frac{h_c\vec{\sigma}\vec{B}h_c}{2M}. \tag{40}$$

Since the chromomagnetic field is due to the light cloud, it is quite obvious that

$$\vec{B} = \frac{C}{2}\vec{S}_{\rm l.c.}, \tag{41}$$

where C is a constant and $1/2$ is introduced for convenience. Then we easily obtain that

$$\Delta M^2(\Omega_c^{(3/2)}) \equiv < \Omega_c^{(3/2)}|\bar{h}_c\vec{\sigma}\vec{B}h_c|\Omega_c^{(3/2)} >= \frac{1}{2}C \tag{42}$$

and

$$\Delta M^2(\Omega_c) \equiv < \Omega_c|\bar{h}_c\vec{\sigma}\vec{B}h_c|\Omega_c >= -C. \tag{43}$$

Here $\Omega_c^{(3/2)}$ is the corresponding member of the baryon sextet with the spin equal to 3/2. As a result, the matrix element of the operator $\mathcal{O}_G$ is given by

$$\frac{1}{2M_{\Omega_c}} < \Omega_c|\mathcal{O}_G|\Omega_c >= - < \Omega_c|\bar{h}_c\vec{\sigma}\vec{B}h_c|\Omega_c >= \frac{2}{3}(M^2(\Omega_c^{(3/2)}) - M^2(\Omega_c)). \tag{44}$$

The mass difference $M(\Omega_c^{(3/2)}) - M(\Omega_c)$ can be determined using nonrelativistic quark model [25] and is ~ 50 MeV.

Consider now the matrix element of the operator $\mathcal{O}_\pi = -\bar{c}\vec{D}^2 c$. The matrix element of this operator is nothing else than the average momentum square of light degrees of freedom. In ref. [26] it is argued that the matrix elements of $\mathcal{O}_\pi$ are essentially independent of the particular hadronic state and are approximately the same for all c containing hadrons. In the absence of better ideas we accept this statement for orientation. If so, one can use the estimate of this matrix element from ref. [21].

The most difficult task in the baryon sector refers to the matrix elements of the four-quark operators. Unfortunately, in this case we do not have such a strong tool as factorization applicable in the case of mesons. In fact, at present we do not know any model-independent approaches permitting one to determine or even to estimate the matrix elements over the baryons. In order to say at least something in this case we have to turn to different models that are not based on first principles and are related to QCD only remotely, if at all. Consequently, the estimates of the matrix elements will be beyond theoretical control.

In the literature analysis of the relevant matrix elements over the baryonic states was carried out only in the nonrelativistic quark model (NQM) [2] and its close relatives, such as the bag model. The former will be discussed at length. Numerically the bag model predictions turn out to be rather close. As a matter of fact, essentially one feature of NQM is crucial – the absence of the spin-spin terms in the matrix elements of $\mathcal{O}_{4q}$ over the triplet baryons (for brevity this clumsy combination of words is substituted by a shorthand 'spin-spin interactions'). The overall normalization of the matrix elements does not matter much since effectively it is adjusted in an *ad hoc* way anyhow (cf. e.g. a 'static' prescription for $M_{\Sigma_c} - M_{\Lambda_c}$ below).

Let us recall some basic features of this model developed by De Rujula, Georgi and Glashow [25] long ago. It is assumed that the hadron consists of 2 (meson) or 3 (baryon) constituent quarks. The quarks in this model have 'effective' masses, e.g. $m_c \sim 1.5$ GeV, $m_u \sim 0.34$ GeV, etc. In order not to mix them with the current quark masses we denote them as m^*. The quarks are described by nonrelativistic spinors and are bound by a nonrelativistic

[2]In NQM non-relativistic limit is assumed for both, c quark and the light ones from the cloud.

potential modified by hyperfine interactions. The hamiltonian is equal to

$$H = L(\vec{r}_1, \vec{r}_2 ...) + \Sigma(m_i^* + \frac{\vec{p}_i^2}{m_i^*}) + \Sigma(\alpha Q_i Q_j + k\alpha_s) S_{ij}. \quad (45)$$

Here m_i^* are the constituent quark masses, $\vec{p}_i$ are the quark momenta, and $\vec{r}_i$ are their coordinates; Q_i are the quark charges, α_s is the QCD coupling constant, and S_{ij} denotes a relativistic hyperfine interaction. The constant $k = -4/3$ for mesons and $k = -2/3$ for baryons. The potential L is a universal quark-binding potential. The QCD interactions due to the gluon exchange are modeled by the Breit-Fermi interaction S_{ij} assumed to have the same form as the electromagnetic Breit-Fermi interaction S_{ij} We refer the reader to the original paper [25] for a detailed discussion of this model and explicit (but rather complicated) expressions for S_{ij}. Using the mass dependence of the hyperfine interactions it is possible to show that in this model the absolute values of the squares of the wave functions of the baryon and the meson at the origin are connected as follows: [25, 27]

$$\frac{|\psi(0)^{(cd)}_{\Lambda_c}|^2}{|\psi(0)_D|^2} = \frac{2m_u^*(M_{\Sigma_c^+} - M_{\Lambda_c^+})}{(M_D - m_u^*)(M_D - M_{D^*})}. \quad (46)$$

Following refs. [4, 5, 6] let us try to extract as much information as we can from the nonrelativistic quark model concerning the 4-quark operators under investigation. As we already noted above, the price will be the appearance in our equations of some parameters which are not connected in any way with the basic QCD, such as a constituent quark mass. Consider, for example, the Λ_c baryon.

We begin from the matrix elements of the operator $\bar{c}\Gamma_\mu c\bar{d}\Gamma_\mu d$. These matrix elements are proportional to $|\psi(0)^{(cd)}_{\Lambda_c}|^2$, the square of the baryon wave function at the origin. More exactly, this is the probability for c and d quarks to meet. The coordinate of the third quark is integrated over. From now on the superscript cd will be omitted. The relation (46) connects the squares of the wave functions in the baryons and mesons [27, 6] and will be systematically applied to make estimates in baryons. Note that in the limit $m_c \to \infty$ we are interested in $2M_D(M_{D^*} - M_D) = \mu_G^2 \times 4/3$ and $M_{\Sigma_c^+} - M_{\Lambda_c^+}$ tends to a constant value. Consequently,

$$|\psi(0)_{\Lambda_c}|^2 \sim \frac{3(M_{\Sigma_c^+} - M_{\Lambda_c^+}) m_u^*}{\mu_G^2} |\psi(0)_D|^2, \;\; |\psi(0)_D|^2 = \frac{1}{12} F_D^2 M_D \kappa^{-4/9}. \quad (47)$$

Recall now [5, 6] that the matrix element of the operator $\bar{c}\Gamma_\mu c\bar{d}\Gamma_\mu d$ over the baryon H_c is equal to

$$\frac{1}{2M_{H_c}} < H_c|\bar{c}\Gamma_\mu c\bar{d}\Gamma_\mu d|H_c >= |\psi_{H_c}(0)|^2(1-4\vec{S}_c\vec{S}_d) \tag{48}$$

where $\vec{S}$ denotes the spin operator. In baryons from the antitriplet – Ξ_c and Λ_c – there is no correlation between the spin of the heavy quark and the spins of the light quarks. Hence the spin part of eq. (48) is zero. Using the expression for $|\psi(0)_D|$ in terms of F_D and M_D we immediately obtain

$$\frac{1}{2M_{\Lambda_c}} < \Lambda_c|\bar{c}\Gamma_\mu c\bar{d}\Gamma_\mu d|\Lambda_c >\sim \frac{1}{4\mu_G^2}(M_{\Sigma_c^+} - M_{\Lambda_c^+})m_u^* F_D^2 M_D \kappa^{-4/9}. \tag{49}$$

In the same way we find for the sextet Ω_c baryon

$$\frac{1}{2M_{\Omega_c}} < \Omega_c|\bar{c}\Gamma_\mu c\bar{d}\Gamma_\mu d|\Omega_c >\sim \frac{5}{6\mu_G^2}(M_{\Sigma_c^+} - M_{\Lambda_c^+})m_u^* F_D^2 M_D \kappa^{-4/9}. \tag{50}$$

Here we assumed that the spin splittings like $M_{\Sigma_c^+} - M_{\Lambda_c^+}$ do not depend on flavor and are the same for all charmed baryons.

The matrix elements of other operators over Λ_c can be expressed via matrix element (49) using the Fierz identities and antisymmetry in color of the baryon wave functions. In particular, using the color antisymmetry of the baryon wave functions we obtain that [5, 6]

$$< \Lambda_c|\bar{c}_i\Gamma_\mu c^j \bar{d}_j\Gamma_\mu d^i|\Lambda_c >=< \Lambda_c|\bar{c}\Gamma_\mu d\bar{d}\Gamma_\mu c|\Lambda_c >= - < \Lambda_c|\bar{c}\Gamma_\mu c\bar{d}\Gamma_\mu d|\Lambda_c > . \tag{51}$$

Similar relations can be found for all other charmed baryons.

Note that if we want to pursue a systematic expansion in $1/m_c$ we must use the static value of the meson decay constant F_D in all expressions above.

5 Static Versus Actual f_Q and Speculations on Higher Order Corrections

Above we have carried out a systematic expansion of the inclusive width in $1/m_Q$ including the contributions of all operators with dimensions up to 6. Let us consider our estimates of dimension 6 four-quark operators more

carefully. As it was already mentioned if we want to have a systematic $1/m_Q$ expansion we must throw away the part of f_D that contains the terms suppressed by powers $1/m_Q$ and use the static meson decay constant F_D in all our estimates. If we want to deviate from this procedure we must take into account the terms of dimension 7 and higher.

A natural question is whether we have to deviate from this procedure. After all, it is reasonable to expect that the contribution suppressed as $1/m_c^4$ will be unimportant. Unfortunately, this is not the case. If we combine the formulae of section 2.5 with the estimates of the 4-quark operators obtained using F_D, we obtain that they give the contribution to the total width of D^+ meson which is negative and $\sim 3\times$ the width of D^0 meson. This is clearly nonsense, meaning that the higher dimensional operators play an important role in the hadronic width of D^+ mesons. Can we model in any way the contribution of the higher-dimension operators? Can we at least argue what they must show up in order to have sensible results?

First, consider the mesons. We shall argue that in the meson case there is at least a set of higher-dimension operators whose effect is to renormalize F_D to f_D.

Indeed, consider the operators appearing in the expansion of the four-quark operators in terms of the operators of HQET. Recall that heavy quark currents like $\bar{c}\Gamma c$ have the following expansion :

$$\begin{aligned}(2M_D)^{-1}\bar{c}\Gamma c = \; & \bar{h}_c \Gamma h_c \\ & +(i/2M_D)(\bar{h}_c\Gamma\hat{D}h_c - \bar{h}_c \overleftarrow{\hat{D}}\,\Gamma h_c) + O(1/m_c^2).\end{aligned} \tag{52}$$

Here $\hat{D}\equiv\gamma^\alpha D_\alpha$. The four-quark operators we are interested in also have a similar expansion,

$$\begin{aligned}\tfrac{1}{2M_D}\bar{c}\Gamma_\mu c\bar{q}\Gamma_\mu q \; & = (\bar{h}_c\Gamma_\mu h_c\bar{q}\Gamma_\mu q \\ & +(i/2M_D)(\bar{c}\Gamma_\mu\hat{D}h_c - \bar{h}_c \overleftarrow{\hat{D}}\,\Gamma h_c))\bar{q}\Gamma_\mu q + O(1/m_c^2).\end{aligned} \tag{53}$$

The matrix element of the operator (53) is naturally expanded in terms of the HQET operators. The matrix element of the first term in this expansion is equal to

$$< D|\bar{h}_c\Gamma_\mu h_c\bar{q}\Gamma_\mu q|D >= F_D^2 M_D/6. \tag{54}$$

We used the Fierz transformation and factorization to estimate the latter matrix element.

By the same token, assuming that factorization still holds for the next terms we obtain that the matrix element of the second term in the expansion (53) is such that

$$\frac{1}{2M_D} < D|\bar{c}\Gamma_\mu c\bar{q}\Gamma_\mu q|D >= \frac{1}{6}M_D F_D^2(1 + 2C_1/M_D + ...). \tag{55}$$

The right hand side of the latter equation is nothing else than the first two terms in the $1/m_c$ expansion of $f_D^2 M_D$.

The above arguments show that it is natural to assume that within factorization the role of the higher dimensional terms is to renormalize F_D to f_D in calculating the 4-quark matrix elements. Unfortunately, the situation is not that simple. First, there are other terms of higher dimensions beyond the ones that appear in the HQET representation of the four-quark operator. Second, even among the operators that appear in this representation there are "nonfactorizable" ones, that can not be reduced to the form of the product of the light quark current – heavy quark current. We can only hope that future investigation will clear the situation.

Note that the need of renormalization of F_D directly follows from the hope that our theory has sense in the charmed family. Indeed, if we use F_D in the estimates of the 4-quark operators in mesons, we obtain that the width of D^+ meson is negative (~ -2 widths of the D^0 meson), a clear nonsense. On the other hand, if we use f_D, we obtain something more reasonable and in accord with the data.

Following the above discussion we conjecture that the effect of the higher dimension terms is to renormalize the leptonic decay constant of the heavy hadron in the estimate of the 4-quark matrix elements from its static to its real value provided that factorization is applicable. We use the 'real world' value of the leptonic decay constant while doing calculations in mesons.

You can view the speculation above (f_D versus F_D) as just an attempt to disentangle the overall normalization of the mesonic matrix elements from that of the baryonic matrix elements, a connection inherent to NQM, if you are not inclined to take it more seriously.

Consider now the case of the baryons. In this case we can not repeat the above calculations – for baryons factorization is not applicable. We can not estimate the contribution of the operators appearing in the second line of (53). We, thus, have to be satisfied with three first orders in $1/m_Q$ expansion, and have no alternative but to use the static constant F_D in our calculations

assuming that here the higher order terms do not play as drastic role as in the meson lifetimes. Quite remarkably, we will see later on that then we do get the experimental pattern of the lifetimes.

We end this section by reiterating that the preasymptotic power terms must be much larger in baryon lifetimes than in meson one, and this can be achieved by using F_D for baryons and f_D for mesons. The difference is formally of higher order in $1/m_c$ – it is necessary to have enhanced terms $\mathcal{O}(m_c^{-4})$ in mesons and suppressed in baryons.

An obvious difference between the baryon antitriplet and mesons is the presence of the strong spin terms in the latter. The same strong spin terms are present in the Ω_c baryon. It is, perhaps, possible to conjecture that an unknown mechanism that distinguishes between the mesons and the antitriplet baryons is somehow connected with the spin terms. If so, a natural consequence might be that the charmed heparin Ω_c has the lifetime around that of D^0, i.e. larger than the lifetimes of Λ_c or Ξ_c^0. It will be very interesting to determine this lifetime experimentally.

6 Numerical Analysis in the Charm Family

We move now to numericals. We shall consider first the case of the charm family. The following numerical values of different parameters are accepted. We use the values $c_+ \sim 0.74, \quad c_- \sim 1.82$ corresponding to the currently accepted value of $\Lambda_{\rm QCD} \sim 300$. We use $f_D \sim 170$ MeV for the 'real world' value of f_D, and $F_D \sim 400$ MeV for the static leptonic decay constant, both normalized at m_c [22, 23]. We choose the normalization point $\mu \sim 0.5$ GeV for the estimates of matrix elements of the four-quark operators. The corresponding value for κ is $\kappa \sim 3.1$. We use the numerical values $m_c \sim 1.4$ GeV, $\mu_G^2 \sim 0.4$ GeV^2, and $\mu_\pi^2 \sim 0.5$ GeV^2.

We also use $M_{\Sigma_c^+} - M_{\Lambda_c} = 400$ MeV, not the real world value $M_{\Sigma_c^+} - M_{\Lambda_c} = 200$ MeV, in the systematic $1/m_c$ expansion, since, as it follows from QCD sum rules [24], this mass difference goes to ~ 400 MeV in the limit of the infinite mass of the heavy quark. (Remember, this is just an overall normalization of the NQM matrix elements, see eq. (46)). For the constituent quark mass we use $m_u^* \sim 0.35$ GeV.

In the naive parton model (the dimension 3 operator $\mathcal{O}_0$) all hadrons have

the same width
$$\Gamma = N_c \eta \Gamma_0 \sim 4.5\Gamma_0. \tag{56}$$
They also have the same semileptonic widths
$$\Gamma_{\rm s/l} \sim 2\Gamma_0. \tag{57}$$
The total width in this model is $\sim 6.5\Gamma_0$.

Consider next the contribution of dimension 5 operators. We begin from the charmed mesons. These operators are spectator blind and contribute in the same way to D^0 and D^+. Using the explicit formulae for the matrix elements of $\mathcal{O}_0, \mathcal{O}_G, \mathcal{O}_\pi$ and the Wilson coefficients C_0, C_G, C_π it is easy to obtain that the dimension 5 terms from $\mathcal{O}_0$, $\mathcal{O}_G$ and $\mathcal{O}_\pi$ shift hadronic widths of D mesons by
$$\Delta\Gamma^{(5)} \sim 2.5\Gamma_0. \tag{58}$$
Let us now consider in more detail the case of D^0 meson. Recall that its width also gets contribution from annihilation mechanism $\sim 0.9\Gamma_0$ [28], and that its semileptonic width is suppressed by a factor $\sim 1/2$ [1, 19]. Then the hadronic width of D^0 meson is
$$\Gamma_{\rm hadr}(D^0) \sim 8\Gamma_0. \tag{59}$$
and its total width is
$$\Gamma(D^0) \sim 9\Gamma_0. \tag{60}$$
Of course, all these estimates have only qualitative character although we certainly are always glad when we see the right tendency. To give a feeling of the uncertainty, note that if we use $m_c \sim 1.35$ GeV, the width of D^0 increases by $\sim 10\%$.

This is all with the D^0 meson, since the 4-quark operators of dimension 6 do not contribute to its width.

Let us parenthetically note that D_s is in the same situation as D^0 and is predicted to have the same width in the limit when we neglect $SU(3)$ breaking effects. A tiny difference between D_s and D^0 is exhaustively discussed in a recent work [29], and we will not dwell on this point here.

For the baryons (except Ω_c) the contribution of dimension 5 operators is due only to $\mathcal{O}_\pi$ operator and is $\sim -0.5\Gamma_0$. The chromomagnetic operator O_G proportional to the correlations between the spin of the heavy quark and

the spin of the light diquark does not show up since the spin of the latter is zero for the baryon antitriplet. For Ω_c we estimated the contribution of this operator in section 4.2.

We now move to dimension 6 operators. The values of the Wilson coefficients $K_i\eta_i$, eq. (28), are:

$$\begin{aligned} K_1\eta_1 &\sim 2.28, \\ K_2\eta_2 &\sim -2.45, \\ K_3\eta_3 &\sim -0.88, \\ K_4\eta_4 &\sim 0.45, \\ K_5\eta_5 &\sim 0.31, \\ K_6\eta_6 &\sim -2.65. \end{aligned} \tag{61}$$

Consider first the charmed mesons. The contribution of the 4-quark operators in the width of D^0 meson is zero. For D^+ meson the contribution of the 4-quark operators is given by

$$\Gamma^{(6)} = 48\pi^2\Gamma_0(\frac{K_1\eta_1}{3} + K_2\eta_2)\frac{f_D^2}{M_D^2}\frac{1}{\kappa^{4/9}}. \tag{62}$$

Here $\kappa^{-4/9}$ takes into account the hybrid logarithms that appear in f_D when we move between m_c and a low normalization point. Substituting the numerical values of the parameters given above and using the "real world" value of the constant $f_D \sim 170$ MeV we obtain that the contribution of the dimension 6 operators into the width of the D^+ meson is

$$\Delta\Gamma^{(6)}(D^+) \sim -4\Gamma_0. \tag{63}$$

For the total and hadronic widths of the D^+ meson we immediately obtain (taking into account the suppression of the semileptonic width mentioned above, and the contributions of $\mathcal{O}_0$ and the dimension 5 operators that are the same as for D^0 meson)

$$\Gamma_{\rm hadr}(D^+) \sim 4\Gamma_0, \quad \Gamma(D^+) \sim 5\Gamma_0, \tag{64}$$

$$\frac{\Gamma(D^0)}{\Gamma(D^+)} \sim 2. \tag{65}$$

This result in an agreement with the experimental pattern, as can be easily seen from Table 1.

This agreement, however, must not be taken too literally. Indeed, in the latter estimate we neglected the difference between M_D and m_c in the denominator of eq. (62). Were we distinguishing between M_D and m_c we would obtain, instead of the factor f_D^2/M_D^2 in eq. (62), the factor $f_D^2 M_D/m_c^3$ that will give a huge contribution of dimension 6 operators, close to $-7\Gamma_0$, reducing the width of D^+ almost to zero. This once again shows the importance of the contribution of higher-dimension corrections. In our case it seems logical to substitute m_c by M_D in the estimates since we use the real world value of f_D. If we used the systematic $1/m_c$ expansion and the static value of leptonic decay constant F_D we would obtain an unphysical negative width of D^+ meson. Alas, the charmed hadrons are too light!

Let us now consider the baryons. Using the explicit expressions for the baryon matrix elements in the NQM it is straightforward to get the following expressions for the matrix elements of the dimension 6 operators:

$$\begin{array}{ll} W_d & \equiv \frac{1}{M_{H_c}} < H_c|\mathcal{L}_d|H_c >= (K_1\eta_1 - K_2\eta_2)\delta\Gamma_B, \\ W_u & \equiv \frac{1}{M_{H_c}} < H_c|\mathcal{L}_u|H_c >= (K_3\eta_3 - K_4\eta_4)\delta\Gamma_B, \\ W_s & \equiv \frac{1}{M_{H_c}} < H_c|\mathcal{L}_s|H_c >= (K_5\eta_5 - K_6\eta_6)\delta\Gamma_B. \end{array} \tag{66}$$

Here we neglected the penguin-type structures. The width $\delta\Gamma_B$ is determined by the following expression

$$\delta\Gamma_B = 48\pi^2\Gamma_0|\psi^2(0)|2m_c^{-3} \sim 48\pi^2\Gamma_0\frac{m_u^* F_D^2 M_D(M_{\Sigma_c^+} - M_{\Lambda_c})}{2\mu_G^2 m_c^3\kappa^{4/9}}. \tag{67}$$

The value of $\delta\Gamma_B$ depends on whether we use F_D and what value of $(M_{\Sigma_c^+} - M_{\Lambda_c})$ is substituted. If the static mass difference, 400 MeV, is substituted, along with the static F_D, we immediately obtain that the contributions of the different 4-quark operators are:

$$\begin{array}{ll} W_d & \sim 28\Gamma_0, \\ W_u & =\sim -8\Gamma_0, \\ W_s & \sim 22\Gamma_0. \end{array} \tag{68}$$

For the contributions of the 4-quark operators into the hadronic widths of the charmed baryons we find

$$\begin{array}{ll} \Gamma^{(6)}_{\rm hadr}(\Lambda_c^+) & = W_d + W_u \sim 20\Gamma_0, \\ \Gamma^{(6)}_{\rm hadr}(\Xi_c^+) & = W_u + W_s \sim 14\Gamma_0, \\ \Gamma^{(6)}_{\rm hadr}(\Xi_c^0) & = W_d + W_s \sim 50\Gamma_0. \end{array} \tag{69}$$

Let us now consider the full baryonic and mesonic widths. Taking into account that the semileptonic widths of the triplet baryons are approximately the same as in the naive parton model (no $\mathcal{O}_G$ contribution) and assembling together operators $\mathcal{O}_0$, $\mathcal{O}_\pi$ and 4-quark contributions we immediately obtain

$$\begin{aligned} \Gamma(\Lambda_c^+) &= 26\Gamma_0 \sim 5\Gamma(D^+), \\ \Gamma(\Xi_c^+) &= 20\Gamma_0 \sim 4\Gamma(D^+) \sim 2\Gamma(D^0), \\ \Gamma(\Xi_c^0) &= 56\Gamma_0 \sim 11\Gamma(D^+). \end{aligned}$$

These results are in a qualitative agreement with the experimental pattern in Table 1.

If instead of the static F_D we used f_D, we would get numbers completely contradicting the experimental pattern and leading to too narrow a span of the lifetimes.

We conclude that the scale of the matrix elements of the four-quark operators must be enhanced in baryons compared to mesons. Within NQM we achieve this enhancement by applying a prescription $f_D \to F_D$. From theoretical point of view the obvious difference between the mesons and the antitriplet baryons is the absence of the interactions between heavy quark spin and that of the light cloud in the latter. Is this the underlying reason for the different behavior of the heavy quark expansion in these two cases? There exists one charmed baryon, namely Ω_c, that has a strong spin-spin interaction like in the mesons. If we use the "static" prescription, as for antitriplet baryons, we obtain that the width of this baryon is $\sim 16\Gamma(D^+)$. If we follow the meson example and use the f_D prescription, we obtain that its width is $\sim 2\Gamma(D^+)$. Depending on the prescription used Ω_c can be both the most short-living and the most long-living among charmed baryons. There is yet no experimental data on the width of this baryon. Such data can be crucial to establishing the QCD mechanism of the hierarchy formation.

Note the main distinctions between our analysis and the analysis carried in refs. [4, 5, 6]. In comparison with the first two works, we used different values of the Wilson coefficients. The ones used in [5] correspond to $\Lambda_{\rm QCD} \sim$ 100 MeV. New values of the Wilson coefficients change in the right way the relative contributions of different four-quark mechanisms (proportional to W_d, W_u, W_s, respectively) in the inclusive widths of the baryons. We also discarded a rule of discarding $1/N_c$ accepted in [5]. Instead, we took into account the contribution of dimension 5 operators. Next, following the authors of ref. [6] we used the relation 46 between the absolute values of the

wave functions of baryons and mesons. Finally, we used the static value F_D for the estimates of the baryonic matrix elements.

In comparison with the analysis [6] we included the hybrid logarithms and the contribution of dimension 5 operators.

7 Estimates for Beautiful Hadrons

The analysis for beautiful hadrons proceeds in exactly the same way as for charmed ones. Moreover, we expect our predictions to be more accurate, since the expansion parameter is ~ 0.1.

The only difference with the charm case is the presence of the phase space factors: $f_0 = 1 - 8r + 8r^3 - r^4 - 12r^2 \ln r$ for the $\bar{c}c$ operator and $(1-r)^3$ and $(1-r)^4$ for $\mathcal{O}_G$ and $\mathcal{O}_\pi$ operators respectively. We use the b-quark mass $m_b \sim 4.8$ GeV and the values $f_B \sim 150$ MeV, $F_B \sim 250$ MeV [22, 23]. The normalization point $\mu \sim 0.5$ GeV, corresponding to $\kappa \sim 5.42$ is chosen. The values of the Wilson coefficients are

$$c_+ = 0.84, \;\; c_- = 1.41.$$

The question of the lifetime hierarchy is especially interesting, since in this case we expect the inverse heavy quark mass expansion to work much better than for the charmed hadrons. All calculations become more reliable. It will be of great interest to see whether the approach becomes quantitative for the beautiful hadrons. Recall that we speculated that f_D is more relevant for mesons while F_D is more relevant for baryons. The difference between f_B^2 and F_B^2 is still quite noticeable – about 40%. The same regularity is expected to show up in the beautiful hadrons. Unfortunately, at this point we do not have any reliable experimental data, so we just consider theoretical predictions that follow from the use of the same rule for beauty as for charm.

Using the values of the parameters given in the beginning of this section, we immediately obtain rather large lifetime differences between different beauty mesons. In particular, we obtain that the lifetimes of charged and neutral B mesons can differ by as much as 3 to 5%. The width of the charged meson $\Gamma(B^-)$ decreases due to the interference effects. On the other hand we obtain that the width of Λ_b can be as much as 10% bigger than the width of the B^0-meson and is approximately equal to the width of the Ξ_b^+ baryon.

The width of Ξ_b^0 baryon is $\sim 3\%$ smaller than the width of B^0 meson. It will be extremely interesting to check these predictions experimentally.

Note that the significant difference with the previous estimate of the beauty lifetimes [5] comes because we use the value of $f_B \sim 1.5$ larger than Voloshin and Shifman, as it follows from the recent QCD sum rule analysis [22].

8 Conclusions

In this talk we addressed the question whether it is possible to 'explain' the observed hierarchy of the hadronic widths in the charmed system in QCD on the basis of the heavy quark expansion. Strictly speaking the charm family seems to lie below or, with luck, at the border of the domain where the expansion may be useful, which makes the task absolutely challenging. We speculated that there is a significant difference in the structure of the preasymptotic expansion in the meson and baryon systems. Terms of higher dimensions show up in a completely different way in mesons and baryons. This situation can be effectively modeled by taking into account the first orders of the preasymptotic expansion (up to $\mathcal{O}(m_c^{-3})$) and by using the real world value of the constant f_D in mesons and the static F_D in baryons. If such a picture really exists the problem of a wide spread of the total widths in the charmed family is solved, at least at the qualitative level.

Unfortunately, the origin of this basic difference between mesons and baryons is still unclear. Since the obvious difference between mesons and the antitriplet baryons is the absence of the spin-spin interaction in the latter (and its presence in the former), it will be extremely important to get a reliable data on the lifetime of the sextet baryon Ω_c. This is the only baryon where we expect that the spin-spin interactions play a major role, as in mesons.

Assuming that the same mechanism works also for beauty hadrons, we listed in the last section some estimates for beauty hadrons. It is extremely interesting to check them in current experiments.

Finally we must stress that our estimates are just provocative. Our task here was , first of all, to formulate the problem of the hierarchy, to see what is known in the current literature about it, and whether it can be solved using available methods. We have speculated how QCD must work in order

to explain the hierarchy. The next question is whether it really works that way.

Acknowledgments It is a pleasure to thank I. Bigi for interesting and stimulating discussions. This work is supported in part by DOE under the contract DOE-AC02-83ER40105.

References

[1] I. Bigi, N. Uraltsev, A. Vainshtein Phys. Lett., B293 (1992) 430.

[2] B. Blok and M. Shifman, Nucl. Phys., B399 (1993) 441, 459.

[3] M. Shifman and M. Voloshin, 1982, see in V. Khoze and M. Shifman, Uspekhi Fiz. Nauk 140 (1983) 3 [Sov. Phys. Uspekhi 26 (1983) 387].

[4] N. Bilić, B. Guberina and J. Trampetić, Nucl. Phys. B248 (1984) 261; M. Shifman and M. Voloshin, Yad. Fiz., 41 (1985) 463 [Sov. J. Nucl. Phys. 41 (1985) 120].

[5] M. Shifman and M. Voloshin, ZhETF, 91 (1986) 1180 [JETP 64 (1986) 698.]

[6] B. Guberina, R. Rückl, J. Trampetić, Z. Phys. C33 (1986) 297.

[7] E. Eichten and B. Hill Phys. Lett., B234 (1990) 511.

[8] H. Georgi Phys. Lett., B240 (1990) 447.

[9] H. Georgi, preprint HUTP-91-AO39;
M. Neubert, preprint SLAC-PUB-6263, 1993.

[10] M. Shifman and M. Voloshin, Yad. Fiz. 45 (1987) 463 [Sov. J. Nucl. Phys. 45 (1987) 292];
H. Politzer and M. Wise, Phys. Lett. B206 (1988) 681.

[11] M. Whitherell, Talk at 16th International Symposium on Lepton-Photon Interactions, Cornell, August 1993.

[12] G. Altarelli and L. Maiani, Phys. Lett., B52 (1974) 351;
M. K. Gaillard and B. W. Lee, Phys. Lett., B52 (1974) 311.

[13] G. Altarelli et al., Phys. Lett, B99 (1981) 141; Nucl. Phys. B187 (1981) 461;
A. Buras and P. Weisz, Nucl. Phys. B333 (1990) 66.

[14] G. Altarelli, Plenary talk at the 1993 EPS Conference, Marseille, July 1993.

[15] I. Bigi and N. Uraltsev, Phys. Lett. B280 (1992) 120.

[16] E. Braaten, S. Narison and A. Pich, Nucl. Phys. B373 (1992) 581;
F. Le Diberder and A. Pich, Phys. Lett, B286 (1992) 147; B289 (1992) 165.

[17] I. Bigi, M. Shifman, N. Uraltsev and A. Vainshtein, to be published.

[18] G. Altarelli and S. Petrarca, Phys. Lett. B261 (1991) 303.

[19] I. Bigi, B. Blok, M. Shifman, N. Uraltsev, A. Vainshtein Talk at DPF-92 meeting, Fermilab, November 1992.

[20] B. Blok and M. Shifman, Nucl. Phys. B389 (1993) 534.

[21] P. Ball and V. Braun, preprint MPI-Ph/93-51.

[22] T. Aliev and V. Eletsky, Yad. Fiz. 38 (1983) 551 [Sov. J. Nucl. Phys. 38 (1983) 936];
V. Eletsky and E. Shuryak, Phys. Lett. B276 (1992) 191;
M. Neubert, Phys. Rev. D45 (1992) 2451; D46 (1992) 1076.

[23] C. Bernard, J. Labrenz and A. Soni, preprint UW/PT-93-06.

[24] E. Shuryak, Nucl. Phys. B198 (1982) 83 .

[25] A. De Rujula, H. Georgi and S. Glashow, Phys. Rev. D12,147 (1975).

[26] A. Manohar and M. Wise, CalTech preprint CALT-68, 1993.

[27] J.L. Cortes and J. Sanchez-Guillen, Phys. Rev. D24 (1981) 2982.

[28] B. Blok and M. Shifman Yad. Fiz. 45 (1987) 841 [Sov. J. of Nucl. Phys., 45 (1987) 522].

[29] I. Bigi and N. Uraltsev, to be published.

Table 1

Experimental Data on Weak decays of Charmed Hadrons

hadron	**lifetime**
D^+	$\sim 10.6 \times 10^{-13}$ s
D^0	$\sim 4.2 \times 10^{-13}$ s
Λ_c	$\sim 2 \times 10^{-13}$ s
Ξ_c^+	$\sim 3 \times 10^{-13}$ s
Ξ_c^0	$\sim 0.8 \times 10^{-13}$ s

Figure Captions.

Fig. 1: The imaginary part of this graph determines the hadronic width of all c containing hadrons in the parton model approximation. The closed circle denotes the effective weak lagrangian.

Fig. 2: The influence of the soft modes. A soft gluon (absorbed by the light cloud of D) is marked by x.

Fig. 3 a,b,c: The diagrams giving rise to $\mathcal{O}_{4q}$ in $\hat{T}$. The crosses mark soft d, u or s quark lines